

**Collusion in Organizations and Management of Conflicts
through Job Design and Authority Delegation[♦]**

by

Yutaka Suzuki

*Faculty of Economics, Hosei University
4342 Aihara, Machida, Tokyo 194-0298 Japan
E-mail: yutaka@hosei.ac.jp*

Abstract

We analyze a principal-supervisor-two agent hierarchy with supervisory efforts, provide a characterization of the equilibrium of the game, and show which regime improves efficiency between the collusion-proof regime and the lateral collusion one, under the assumptions that the principal is less informed, and that the side-trade is costly. By coping with the trade-off between the value of information vs. either the cost of the collusion incentive constraint (in the collusion-proof regime) or the rent-seeking cost (in the equilibrium collusion one), for some parameters, the principal may want to adopt the collusion-proof contracts, and for other parameters, let collusion happen in equilibrium. As a characterization result, we derive the low-powered job for the agent and the high-powered job for the supervisor in each of the two regimes. Finally, we show how the allocation of real authority is endogenously determined, and interpret it from the viewpoint of the centralized vs. decentralized firms.

Key Words Collusion-Proof Contracts, Equilibrium Vertical Collusion, Equilibrium Lateral Collusion, Authority Delegation, Centralized vs. Decentralized Firms.

JEL Classification D23, D74, D82

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1, Introduction

In hierarchical organizations where a supervisor(s) monitors agents for the benefit of the principal, manipulation of information may arise when agents and supervisor(s) collude to conceal the relevant information from the principal. This paper addresses this problem within the framework of triangular or multilateral agency relationships, where participants may contemplate side contracting. Collusion means that within a group of participants, a coalition forms a strategic alliance at the expense of the rest of the group.

The research has addressed the possibility of supervisor-agent coalition formation within a three-tier hierarchy, where the principal may wish to monitor an agent and so hires a supervisor to perform the task effectively. However, the supervisor may be often purely self-interested, and willing to accept a payment (bribe) from the agent in return for misreporting his observations. The manipulation of information through the collusion between the supervisor and the agent may bring about a large loss for the organization, since the ‘wrong’ task assignment may be realized. Hence, the principal may exercise the option to create collusion-proof contracts to deter the supervisor’s misbehavior. This is a familiar result in the collusion literatures following the model of Tirole (1986).

The focus of this paper is to show that under the principal-supervisor-*two agents* hierarchy *with supervisory efforts*, in some cases, the collusion-proof contracts may be the second best solution, but in the other cases, allowing the possibility of collusion and promoting cooperation among a subgroup of actors may be welfare enhancing. Further, a self-interested supervisor is unlikely to be of value to the principal due to the threat of manipulation of information through the collusion. What we do in this study is to investigate the conditions under which each solution is selected as the second best option, characterize the nature of the incentive schemes, and relate the optimal solutions to the problem of authority delegation in organizations, especially from the viewpoint of formal and real authority introduced by Aghion and Tirole (1997).¹

To illustrate the problem, this paper studies the instance where the agents and the supervisor by observing the types of agents obtain accurate information about the types, but the principal does not. Let us present one concrete example in the real world. In product development organizations, where the product manager and the employees are engaged in development activities, the information on how much it costs for an employee group to perform a particular activity level is often privately observed among the field members but is not verifiable. Also, the action combination, particularly the level of the supervisor’s effort is not observed by a third party. That is, how much cost reduction was made due to the supervisor’s effort is not observable ex post. For these reasons, even though the total cost may be observable, the individual cost contribution (the cost allocation) may not be observable even ex post. This corresponds to the accounting practice whereby, while the total cost is observable, the allocation of the individual costs is often not verifiable, because the cost information of the agents is privately observed only among the project participants and the existence of a common factor such as the supervisor’s effort makes it difficult for the auditor to calculate, determine and verify the individual cost contribution at the accounting level. In numerically terms, the equation with the total cost $100 = 150$ (high cost type’s activity cost) $- 50$ (the supervisor’s cost reduction) $= 120$ (low cost type’s activity cost) $- 20$ (the supervisor’s cost reduction) holds. Thus, even though the total cost of 100 is observable and verifiable, if the supervisor’s cost reduction is not observable, the agent’s type cannot be deduced ex post. This will bring about a serious informational problem in the cost management of product development organizations.²

¹ For a recent paper that has a connection with Aghion and Tirole (1997), see Aghion-Dewatripont-Rey (2002). For a recent survey and insights on authority in organizations, see Dewatripont (2001).

² For example, in a multiple agent setting, Rajan (1992) shows that cost allocation schemes can be valuable in reducing

In addition to this information (monitoring) problem, in important projects, the employees may have the incentive to pursue a good task, since it not only provides large and immediate monetary rewards but after the completion of the current project guarantees that they receive a large future prize that often takes the form of a promotion or reputation enhancement. Especially, when the agents expect a zero-sum game structure in future promotions, depending on the current task assignment, they may have a strong incentive to persuade the supervisor to form a side contract where the agent pays the supervisor to reveal or conceal the relevant information to or from the principal. This structure, where in every (ex ante) state of nature the *promoted* agent can obtain a net additional utility (*prize*), creates the adverse selection problem central to our model.

In vertical relationships where the common supervisor reports to the principal the type of agents and the task assignment is determined by his report, a high possibility exists that there may be collusion between the supervisor and one agent, which is potentially damaging to the excluded agent. This will lead to fierce competition between the two agents to form a vertical collusion contract (side contract). The total effect of allowing this competition may be, on one hand, to reduce overall efficiency, but on the other hand, it may be less costly compared with the option to deter the competition by means of a collusion-proof scheme, from the perspective of the organization. Even when this competition reduces the overall efficiency compared with the collusion-proof regime, if the possibility of horizontal (we shall call it 'lateral' here) collusion exists among competing agents that aim to internalize the existing negative externalities, it may largely reduce the negative attributes of competition, and so enhance overall welfare.

At a theoretical level, we could ask the following question: Suppose that the principal designs one innovation task and one routine task for two agents (one is efficient and the other is inefficient) and one task (one activity level) for the supervisor, but does not know which task fits better with which agent's type (e.g. agent's ability, cost). Since the supervisor knows the abilities (types) of the agents, the principal wants him to recommend which task should go to which agent. Should the principal design the tasks (jobs) so as to induce the supervisor and the agents to give up collusions or so as to allow them to collude? Then, we should note that even though the principal "designs" these three tasks and cannot delegate "designing tasks" itself to other parties, the importance of "the task assignment problem" remains. It is because the task assignment decision itself can be interpreted as a *non-contractible action* and determined as an equilibrium outcome of hidden-gaming between the supervisor and the two agents, and therefore it may be in the interests of the principal endowed with formal authority, when he/she is *uninformed* of which task fits better with which agent's type, to "grant" real authority, i.e., the authority on which to *really* make the decision on the task assignment to the other parties who are *informed*.

We analyze such questions in an agency model with collusion. In section 2, we present the model, and characterize the first best actions. Then, we check the logic under which the competition for vertical collusion as described above may occur. In section 3, we analyze the three types of second best regimes - collusion-proof, vertical collusion, and lateral collusion - and then characterize the nature of the solutions. In section 4, we compare the second best efficiency of each regime under the conditions for the exogenous parameters. In section 5, we provide interesting interpretations of the model from the viewpoint of organization structure (centralization vs. decentralization) and the delegation of real authority on task assignment. Section 6 suggests applications. Section 7 concludes this paper.

the agents' ability to *collude to misreport* the type information. In the setting for this paper, while the total cost is observable (verifiable), the allocation of the individual costs (cost allocation) is not observable (verifiable) ex post. Hence, Rajan's cost allocation schemes do not work.

1.1 Related literature

Though this paper's framework is based on Tirole (1986, 1992) and Laffont and Tirole (1991), the results and the underlying intuition are also close to Milgrom (1988), which states that efficient organization design counters influence activities by limiting the discretion of decision makers, especially for those decisions that have large distributional consequences, but that are otherwise of little consequence to the organization. The main departure from Milgrom is that our paper includes a more explicit model of the collusion game played by the supervisor and agents, while Milgrom's paper deals with more general but less modeled influence activities. Modeling the collusion game explicitly gives us distinct predictions as to collusion proof vs. equilibrium collusion, decentralization and delegation, applications to political economy, and how various forms of collusion lead to inefficiency in the three-tier, four-agent contracting problem.

Segal (1999) generally examined contracting with externalities in the class of principal-many agents problems. In contracting with externalities, when some contracts (e.g., bilateral) are feasible, while the grand contracts are infeasible, externalities may not be fully internalized. As a result, the so-called Coase theorem will not hold even with full information. That is, limitations on the set of available contracts, for instance, bilateral contracting will lead to inefficient outcomes. Segal clarifies the mathematical structure essentially shared in many applications, including vertical contracting, exclusive dealing, and takeovers. Though this general idea is certainly also applied to this paper, his paper considers the "Offer Game", where the principal makes take-it-or-leave-it (TIOLI) offers to agents, while this paper considers the "Bidding Game", where agents make take-it-or-leave-it (TIOLI) offers to the supervisor, given the incomplete grand contract offered by the principal.

Dewatripont and Maskin (1995) analyzed the "Soft Budget Constraint" problem in a dynamic adverse selection model. If the principal can commit *not* to renegotiate (not to bail out) at the end of the first period, only good types apply for the project and the first best can be attained. But, if ex post renegotiation is possible, that is, the principal cannot commit not to bail out, both types apply (the bad types mimic the good ones). This brings about inefficiencies due to *inability* to commit. In our paper, if the supervisor can commit *not* to collude with the bad type, the first best contract can be implemented. Otherwise, the supervisor may collude with the bad type and manipulate the information collusively. Then, since the original contract is no longer collusion-proof, it is necessary for the principal to contrive optimal arrangements and we compare three schemes.

2. Model

2.1 Setup

We consider an organization, consisting of four risk neutral players; a principal (P), a supervisor (S) and two agents (1 and 2), with P taking the top position, 1 and 2 the bottom and S the middle. The principal owns the idea for the project to be undertaken, but the actual implementation of the project is delegated to two of the rest of the members, *i.e.*, the *development team* consisting of the supervisor and one agent.

There is one important and innovative task in the project and either of two agents is assigned to it. Two agents are the same in that both of them still own an asset (skill) indispensable for the project, but the innovative task is only one and indivisible. The required level of the innovative task is written in the contract. An agent assigned to the innovative task supplies an effort $a_i \in \mathbb{R}_+$ that produces output $y_i = f(a_i)$. We assume that f is concave so that $f'(a_i) > 0$ and

$f''(a_i) < 0$ for all a_i . The cost to produce y_i for organization C_i depends upon the ability of the

assigned agent. This ability is represented by the cost parameter θ_i , which takes the value of either k or $-k$. If $\theta_i = k$, the agent's ability is high (we refer such an agent interchangeably either as an agent of high ability type or of type k), while if it is $-k$, his ability is low.

For simplicity, we assume that exactly one agent has high ability and the other is low, and that both states occur with the equal probability. It can be summarized by the following assumption:

$$(A.1) \text{ Perfect Correlation: } \theta = (\theta_1, \theta_2) = \begin{cases} (k, -k) \text{ with probability } 1/2 \\ (-k, k) \text{ with probability } 1/2 \end{cases}$$

The supervisor takes the middle position within the firm. His role is twofold: collecting information about the overall situation (the state) and reporting it as $\hat{\theta}$ (truthfully or non-truthfully) to the principal, and helping agents in the development process by contributing his own effort $e \in \mathbb{R}_+$. The latter action will reduce the cost of the task. To be specific, the cost of performing task i by type θ_i agent when he chooses effort a_i and the supervisor chooses effort e , $C_i(a_i, e; \theta_i)$, takes the following specific form;

$$C_i(a_i, e; \theta_i) = (\beta - e - \theta_i) a_i \quad (1)$$

where β is a positive constant sufficiently large so that the cost C_i is positive for any relevant values of e and θ_i . In view of A.1, the development cost is expressed as;

$$C(a, e; \theta = k) = (\beta - e - k) a \quad (2)$$

where a represents the activity level that the agent of high ability ($\theta = k$) should choose.

Providing effort of e costs the supervisor $\psi(e)$ with $\psi'(e) \geq 0, \psi''(e) \geq 0$ for all e .

We assume that state θ is observable by the agents and the supervisor, and so are common knowledge to these parties, but the principal does not observe it. An individual agent's choice of input a_i (or, equivalently, individual output y_i), as well as the cost of the production team C , are observable and verifiable by all parties. We also assume that the principal cannot observe (and hence cannot verify) the supervisor's effort e . It follows that the principal, with the *ex post* information of (a, C) , will not be able to identify which agent is of high ability *if the supervisor is willing to spend extra resources to conceal the information*. The reason is as follows.

Let us suppose that the supervisor reports false information about the state. Then, the total cost will increase up to $C'(a, e; \theta = -k) = (\beta - e + k) a$ (2')

if the supervisor does not exert any extra effort. Subtracting (2) from (2'), we obtain the cost overrun $C' - C = 2ka > 0$. On the other hand, if he exerts the extra effort $\Delta e = 2k$, the cost will be

$$C(a, e + \Delta e, -k) = (\beta - e - \Delta e + k) a = (\beta - e - k) a \quad (3)$$

and the same cost level as (2) will be achieved. The point is that the supervisor's extra effort contributes to a reduction in the cost without being observable by the principal. This is the "Hidden Action" aspect examined in this paper, while the unobservability of θ is the setting of "Hidden Characteristics".

The principal designs the main contract, which describes the verifiable innovation activity level a and the cost target C . Then, the principal assigns the innovative task a to the agent who is identified as *efficient* by the supervisor's report $\hat{\theta}$. The output $f(a)$ is fully extracted by the

winning agent, from which the supervisor's wage W_S is subtracted conditional upon the cost target C being achieved.³

It is assumed that the principal can commit to this contract *ex ante*, which corresponds to an organizational form, implying a collusion-proof scheme or equilibrium collusion scheme, as explained later.

Given the main contract by the principal, the agents and supervisor can form a collusive agreement by means of a side contract. Side contracts specify the amount of side transfer t_i to the player $i = 1, 2, S$, which depends on the agent's choice of effort a_i , and the supervisor's report $\hat{\theta}$.

Now, following Tirole (1992), we shall define a transaction technology that specifies the transfer received by the recipient. We assume that they can transfer income equivalent to one another at rate $\lambda \in [0, 1]$. That is, if an agent transfers t to supervisor, he receives λt . Where $\lambda = 0$, this corresponds to a full deadweight loss in the transaction and yields the no-collusion case. Where $\lambda \in (0, 1)$, this corresponds to a positive deadweight loss and $\lambda = 1$ to no deadweight loss in the side transaction.

We assume that this side contract can arise in a collusion game either where two agents simultaneously make a take-it-or-leave-it offer to the common supervisor for side contracting, or where two agents negotiate a collusive agreement between them.

2.2 Objective of Agents: "Private Benefit", including Such Things as Future Promotion Possibility, Relative Status (Rank) Consideration, and Reputation Enhancement

The winning agent captures the full monetary return from the output $y = f(a)$, while the losing agent receives nothing. They receive/spend a side transfer through a collusive contract, and pay effort costs. In addition, they receive an extra reward that is outside the control of the current players ϕ . It may be best interpreted as the possibility for a future promotion and reputation enhancement within the organization. Based on the usual practice of the organization, we assume that just one of the two agents will be promoted to a higher position with a larger payoff. For simplicity, we assume that the possibility of promotion is determined by the activity level a_i for the project, and that the agent who was assigned to the innovative task and made the larger effort (activity) will be the one to be promoted. With promotion, agents will receive the extra payoff of $\phi_i(a_i)$ as a private benefit. We assume that the amount of extra gain for the winning agent is equal to the amount of extra loss (negative private benefit) for the losing agent⁴. Theoretically, this means a zero-sum structure and so the existence of a *negative externality* between agents.

³ This can be interpreted as a contract in which the supervisor is asked to announce the type information $\hat{\theta}$ and each agent receives a bundle $(a(\hat{\theta}), W(\hat{\theta}))$ on the basis of the supervisor's announcement. Thus, this is not equivalent to so-called *Direct Revelation Contract* in that only the supervisor sends a message to the principal. We assume that if the agent receives and accepts the contract, he is forced to produce level a .

⁴ Another interpretation is that agents care about self-esteem arising from a consideration of the relative status (rank). The higher performing agent gains a positive utility from the higher social esteem, while the lower performing agent receives some disutility by being ashamed of occupying a lower status (rank). It would be empirically the case that the engineers exhibit significant concerns about relative status (rank). When we integrate into the model such human emotions as self-esteem and pride or shame generated by human interactions, the consideration of relative status (rank) could be shown as $\mu a - (-\mu a) = 2\mu a$, since a higher (lower) status generates a positive (negative) net feeling

2.3 Timing and Contract

Period 1:

- (a) Principal designs the main contract, which describes the innovation activity level a and the cost target C and offers it to two candidate agents and a supervisor. If the supervisor and agents accept the contract, the game proceeds to (b).
- (b) Supervisor and two agents are aware of the state (θ_1, θ_2) , but the principal is not.
- (c) Agents and supervisor engage in a collusion game to reach a side contract.
- (d) Supervisor reports (sends a message) $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2)$ to the principal. After hearing it, the principal assigns the innovation task to an agent.
- (e) The project starts. Agents and supervisor choose their effort levels. Output $f(a)$ and cost C are realized and observed by all parties.
- (f) The principal's main contract as well as side contracts are implemented.⁵ All transfers are made according to the main and side contracts.

Period 2:

Promotion within the organization is implemented on the basis of the task assignment in Period 1, and agent $i = 1, 2$ receive the extra payoff of $\phi_i(a_i) = \mu a$ if $a_i = a$ and $-\mu a$ if $a_i = 0$.⁶

2.4 The Payoff Functions of Players

Here, we summarize the payoff functions of the two agents and the supervisor.

$$U_i = W_i(a_i) + \phi_i(a_i) - C_i(a_i, e; \theta_i) + t_i \quad i = 1, 2 \quad a_i \in \{0, a\}$$

$$U_s = W_s(C) + t_s - \psi(e)$$

W_i and ϕ_i $i = 1, 2$ are the monetary reward and the non-monetary private benefit. These terms in the payoff function of the agent capture the idea that each agent takes into consideration the extra gain/loss (private benefit) $\phi_i \in \{\mu a, -\mu a\}$, in addition to the monetary return W_i . But since ϕ_i is a

$\mu a(-\mu a)$. For the literature that incorporates a psychological consideration, see. e.g., Kandel and Lazear (1993).

⁵ This timing of the side transfers will bring about the problem of *the enforcement of side contracts*. For example, we suppose that arbitrators or mediators such as a banker or Japanese restaurant can enforce this type of side contract. Then, $1 - \lambda$ can be interpreted as representing the (unit) cost for enforcing a collusive side contract. Alternatively, we could also consider the enforcement mechanism in terms of information sharing, especially, detailed observation regarding the deviating behavior from the agreement. We follow the enforceable side contracts approach, not the self-enforcing one. As for the discussion, see Tirole (1992).

⁶ The term μa corresponds to the "private benefits" in the contract theory literature. See, e.g., Aghion-Bolton (1992), Dewatripont-Maskin (1995), and Hart-Holmstrom (2002).

non-verifiable variable, the parties cannot contract on this variable *ex-ante*, that is, a long-term contract is not possible. The third term $C_i(a_i, e; \theta_i)$ represents the cost that the agent bears for the assigned task, given the supervisor's effort e and the cost type (ability). The fourth term represents the side payments that the agent gives or takes in order to collude with the other players. A positive (negative) number implies that he receives (pays) the side payment. With regard to the payoff of the supervisor, the first term W_s is the revenues guaranteed by the initial (main) contract and the second term t_s is from the side contract. Since we assume that side payments accompany the deadweight cost (a kind of transaction cost), t_s means the amount of the side payment that the supervisor *actually receives*. The third term is the cost of effort e .

2.5 First Best: Collusion-Free Problem

We characterize briefly the first best optimal contract when informational asymmetry does not exist. It maximizes the total surplus, with no incentive constraint.

Let $(a^F(k), e^F(k))$ be the first best actions for a parameter k when θ is public information. It is the solution where $(a^F(k), e^F(k)) = \arg \max_{\{a, e\}} f(a) - (\beta - e - k)a - \psi(e)$

Then it satisfies the first-order conditions

$$f'(a^F(k)) = \beta - e^F(k) - k \quad (4) \quad \text{and} \quad a^F(k) = \psi'(e^F(k)) \quad (5)$$

FOC (4) means that the marginal product of agents' inputs a equals each agent's marginal cost, given the supervisor's effort. We assume the condition $f'(0) > \beta - k$, which assures $a^F(k) > 0$ for any $k > 0$. (5) means that by marginally increasing the supervisor's effort, the marginal savings of the cost equals the marginal disutility of the supervisor's effort. (4)- (5) characterize the marginal incentives that generate the first best maximum surplus.

2.6 The Collusion Problem

The revenue from the output $f(a)$ is fully extracted by the winning agent, from which the supervisor's wage W_s is taken away on the condition that the cost target C is achieved. Since the supervisor's effort cost is $\psi(e)$ under the cost target C , the winning agent obtains

$W_A = f(a) - \psi(e)$ as the gross of the activity cost. This can be explained as follows. The principal and the winning agent bargain individually with the output generated from the development activities. Due to the competitive pressure on the principal, the winning agent has all the bargaining power for the distribution of output $y = f(a)$ and so he obtains all the revenue.

Now, under the sub-game played by the supervisor and the two agents, which is induced by the first best contract, under what conditions does the gain from collusion among the supervisor and the high-cost (low-ability) agent occur?

2.6.1 Incentive for Collusion between Supervisor and High-Cost (Inefficient) Agent

First, we check the incentive of the high-cost agent ($-k$). Under the above setting, the gross utility (monetary and private benefits) when he is assigned to **the innovative task** a is

$$\begin{aligned}
U(a, e + \Delta e; -k) &= f(a) - \psi(e) - (\beta - e - \Delta e + k)a + \mu a \\
&= \underbrace{f(a) - \psi(e) - (\beta - e - k)a}_{\text{monetary benefit}} + \underbrace{\mu a}_{\text{private benefit}}
\end{aligned}$$

and the utility (monetary and private benefits) when he is assigned to **the routine task 0** is

$$U(0; -k) = \underbrace{-\mu a}_{\text{private benefit}}$$

Thus, the difference (the payoff spread before the side transfer to the supervisor is made) is

$$V_2 = U(a, e + \Delta e; -k) - U(0; -k) = f(a) - \psi(e) - (\beta - e - k)a + 2\mu a$$

where $2\mu a = \mu a - (-\mu a)$ is the difference in *private benefit* that the agent generating the output a at the innovative task obtains. Independent of the state $\theta \in \{k, -k\}$, when an agent is assigned to the task a and performs it, he can get the private benefit $2\mu a$ as a prize. Thus, V_2 is the gross payoff increase that a high-cost agent can obtain if he is assigned the innovative task with the high activity level a . This becomes the potential source that generates the incentive to manipulate the information collusively.

The net collusive gain (the joint payoff) to the supervisor and the bad-type agent (high-cost, low ability) is, from the viewpoint of the bad type agent's payoff,

$$\Delta_2 = V_2 - (1/\lambda)[\psi(e + \Delta e) - \psi(e)], \quad \text{where } \Delta e = 2k \quad (6)$$

The second term means the cost of the supervisor's extra effort $\Delta e = 2k$ that the bad (high-cost) agent must compensate for, due to the observability (verifiability) of the cost C . If Δ_2 is positive, the bad (high-cost) agent will have an incentive to mimic the good agent collusively with the supervisor.⁷

Hence, the Collusion Incentive Constraint (CIC) for the bad (high-cost) agent is that

$$\begin{aligned}
\Delta_2 &= V_2 - (1/\lambda)[\psi(e + \Delta e) - \psi(e)] \\
&= f(a) - (\beta - e - k)a - \psi(e) + 2\mu a - (1/\lambda)[\psi(e + 2k) - \psi(e)] \leq 0 \quad (7)
\end{aligned}$$

When this inequality holds, the bad (high-cost) agent does not try to collude with the supervisor, that is, the incentive for collusion between the supervisor and the bad (high cost) agent does not exist. Thus, if the collusion incentive constraint for the bad (high cost) agent is satisfied (non-binding), i.e., $\Delta_2 < 0$ at the first best allocation, in other words, there is no conflict of interests in that the bad agent strictly prefers the routine task 0 to the innovative task a , and vice versa, then the first best efficiency can always be implemented.

Now, when do the situations where both agents compete for the good (innovative) task occur?

It is when the following condition for a subset of parameters holds:

$$\Delta_2 > 0 \text{ evaluated at } (a, e) = (a^F(k), e^F(k))$$

Lemma1:

The collusion incentive constraint (7) for the high cost (bad type) agent is *not* satisfied at the first best actions if and only if

⁷ We find that "cost observability" significantly reduces the gain from collusion, and thus decreases the source of inefficiency.

$$\begin{aligned}\Delta_2 &= V_2(a^F, e^F) - \frac{1}{\lambda} [\psi(e^F + \Delta e) - \psi(e^F)] \\ &= \underbrace{f(a^F) - \psi(e^F) - (\beta - e^F - k)a^F}_{\text{monetary benefit}} + \underbrace{2\mu a^F}_{\text{private benefit}} - \underbrace{\frac{1}{\lambda} [\psi(e^F + 2k) - \psi(e^F)]}_{\text{cost compensation for the supervisor}} > 0\end{aligned}$$

In this case, the first best cannot be achieved.

In this case, *the promise of promotion as a common prize* strongly induces the agents' ex ante competing behavior, causing them to behave collusively with the supervisor. As a result, inefficiencies exist that are impossible to remove.

Moreover, the net collusive gain (the joint payoff) to the supervisor and the good type (low-cost, high-ability) agent is, from the viewpoint of the good type agent's payoff,

$$\Delta_1 = V_1 = f(a) - (\beta - e - k)a - \psi(e) + 2\mu a$$

since there is no supervisor's extra effort cost for the good type to compensate as the bad (high-cost) agent does. So we have:

Lemma2: $\Delta_1 - \Delta_2 = \frac{1}{\lambda} [\psi(e + 2k) - \psi(e)] > 0$

In this section, we showed that the first best contract might result in the third best outcome under the threat of collusion. The standard agency theory shows that, under the first best contracts in the case of risk-averse agents, the expected profit of the principal falls into the third best one *because of the large risk compensation*. Then, the principal marginally changes the incentive contracts, and tries to increase her expected payoff. As a result, the first best surplus cannot be attained. Similarly in this analogy, in the next section, we shall investigate the mechanisms that improve the total surplus of the two agents, which will result in the second best efficiency.

3. Equilibrium: Can Collusion Enhance Efficiency as a Second Best Mechanism?

3.0. Overview

A main contract (a, e) is classified into two major categories by the nature of the equilibrium that the main contract induces in the collusion (side contracting) game.

Suppose that the equilibrium main contract is such that no agent is better off by any side contract which guarantees the supervisor at least the utility from the truth telling report $\hat{\theta} = \theta$. Then, the equilibrium side contract offers are $\Delta_1 = \Delta_2 = 0$. We call this a 'Collusion-Proof Regime'. Next, if at least one agent is better off, then the winning side contract offer is the maximum of the surplus which the losing agent would enjoy from his side contract. The maximum value goes to the supervisor from the winning agent. We call this an 'Equilibrium Vertical Collusion Regime'. We can also consider the possibility of collusion (side contracting) between the two agents, with the threat of 'Equilibrium Vertical Collusion'. We refer to this as an 'Equilibrium Lateral Collusion Regime', and investigate how it works.

3.1 Collusion-proof Regime

Here, we assume that it is optimal for the principal to structure incentive schemes so as to prevent collusion in equilibrium. In Section 4, we show that *collusion proofness* (implementing a collusion-proof contract) is optimal only for a subset of parameters.

To prevent collusion between the supervisor and the high-cost agent when $(\theta_1, \theta_2) = (k, -k)$, the collusion incentive constraint for them must be satisfied. Hence, the problem is as follows.

Collusion Proof Problem: CP

$$\text{Max } f(a) - (\beta - e - k)a - \psi(e)$$

$$\{a, e\}$$

$$\text{s.t. } \Delta_2 \leq 0 \Leftrightarrow V_2 = U(a_1, e + \Delta e; -k) - U(0; -k) \leq (1/\lambda) [\psi(e + \Delta e) - \psi(e)]$$

where $\Delta e = 2k$. Note that in equilibrium $\Delta_1 = \Delta_2 = 0$ and the supervisor makes the truth telling report $\hat{\theta} = \theta$.

Let $\{a^{CP}, e^{CP}\}$ maximizes the total surplus $TS = f(a) - (\beta - e - k)a - \psi(e)$ for parameters k, λ, μ within the class of collusion-proof regimes $\Delta_2 \leq 0$. In order to derive the property of (a^{CP}, e^{CP}) , we present and analyze the Kuhn-Tucker conditions.

A main contract (a, e) is a solution to the collusion-proof regime if and only if there exists a Kuhn-Tucker multiplier ξ such that

$$\left. \begin{aligned} f'(a) - (\beta - e - k) &= \xi \frac{\partial \Delta_2}{\partial a} \\ a - \psi'(e) &= \xi \frac{\partial \Delta_2}{\partial e} \end{aligned} \right\} \Leftrightarrow \left(\frac{\partial TS}{\partial a}, \frac{\partial TS}{\partial e} \right) = \xi \left(\frac{\partial \Delta_2}{\partial a}, \frac{\partial \Delta_2}{\partial e} \right)$$

and $\xi \geq 0, \Delta_2 \leq 0, \xi \Delta_2 = 0$ (Complementary Slackness Condition), where

$$\Delta_2 = V_2 - (1/\lambda)(\psi(e + \Delta e) - \psi(e))$$

$$= \{f(a) - (\beta - e - k)a - \psi(e) + 2\mu a\} - (1/\lambda)(\psi(e + 2k) - \psi(e))$$

$$\text{and } \begin{cases} \frac{\partial \Delta_2}{\partial a} = f'(a) - (\beta - e - \Delta e + k) + 2\mu = f'(a) - (\beta - e - k) + 2\mu \\ \frac{\partial \Delta_2}{\partial e} = a - \psi'(e) - \frac{1}{\lambda}(\psi'(e + 2k) - \psi'(e)) \end{cases}$$

From the FOCs, we have:

$$MRS_{ae} = \frac{\partial TS}{\partial a} / \frac{\partial TS}{\partial e} = \frac{\partial \Delta_2}{\partial a} / \frac{\partial \Delta_2}{\partial e} = MRT_{ae} \quad \dots (*)$$

Equivalently, we have the following formula on the optimal job design.

$$\frac{\partial TS}{\partial a} / \frac{\partial \Delta_2}{\partial a} = \frac{\partial TS}{\partial e} / \frac{\partial \Delta_2}{\partial e} \quad \dots (**)$$

The interpretation is as follows. In both sides of (**), the numerator is the increase in total surplus through a unit increase in the activity/effort, and the denominator is the increase in the tightness (relaxedness) of the collusion incentive constraint similarly through a unit increase in the activity/effort. The increase in total surplus divided by the increase in the tightness (relaxedness) of the collusion incentive constraint means the *effective* marginal increase in total surplus through a unit increase in the activity/effort. The formula (**) therefore says that, at the optimum, the effective marginal increase should be equalized between both a and e . We can describe the equilibrium structure of the collusion proof contract.

Proposition 1: The optimal collusion-proof solution is as follows.

For the activity level a^{CP} , there exists a $\bar{k} > 0$, such that

- (a) For $0 < k < \bar{k}$, we have a low-powered job design $0 < a^{CP} < a^F$.
- (b) For $k \geq \bar{k}$, we have the first best job design $a^{CP} = a^F$.

For the supervisory effort e^{CP} , we have a high-powered job design $e^{CP} \geq e^{FB}$ for all $k > 0$.

Case (a) says that a^{CP} of the collusion-proof solution is intermediate between 0 and a^F , and is thus low-powered. While on the other hand, e^{CP} is greater than e^F , and is thus high-powered.

Proof: See Appendix 1

The virtual cost of the good (low cost) agent is $(\beta - e - k) + \xi(\partial\Delta_2/\partial a)$. If the principal adopts the first best job design, she will obtain a *second-order gain* through changing the optimal behavior. However, the cost through the collusion incentive constraint (CIC) is *discretely large*, in other words, the collusion incentive constraint (CIC) will be violated. The low-powered job design $a^{CP} < a^F$ is better from the viewpoint of efficiency. That is, though the principal certainly fails to make the mechanism *less responsive to information*, she can *reduce the conflict for collusion discretely*. This latter effect dominates for relatively small differences ($k < \bar{k}$) between the agents' types.

Now, λ implies the ease of collusion, and μ leads to a strong desire for collusion. An increase in these parameters will tighten the collusion incentive constraint (CIC). We can obtain the following comparative statics results.

Corollary 1:

- (a) As μ is greater, the optimal collusion-proof solution $\{a^{CP}, e^{CP}\}$ becomes smaller.
- (b) As λ is greater, the optimal collusion-proof solution $\{a^{CP}, e^{CP}\}$ becomes smaller.

Proof: See Appendix 2

3.2 Equilibrium Vertical Collusion

In the second step, we *assume* that it is optimal for the principal to structure incentive schemes so as to allow (induce) collusion in equilibrium *between two parties* consisting of the supervisor and either of the two agents. That is, the principal lets two agents compete for vertical collusion

with the common supervisor. Then, when $(\theta_1, \theta_2) = (k, -k)$, agent 2 (bad-type agent) is willing to pay the gross value $V_2 = U(a_1, e + \Delta e; -k) - U(0; -k)$ to the supervisor, which is in the net value $\Delta_2 = V_2 - (1/\lambda)[\psi(e + \Delta e) - \psi(e)]$.

Then, $\lambda\Delta_2 = \lambda V_2 - [\psi(e + \Delta e) - \psi(e)]$ denotes the side payment (bribe) that the supervisor must receive from the good (low-cost) agent, in order for him to tell the truth, when the coalition incentive constraint of the bad (high-cost) agent is not satisfied, that is, the slack $\Delta_2 > 0$ exists, and he makes the maximum bid Δ_2 to the supervisor.⁸ $\lambda\Delta_2$ is the value for the supervisor of such a side payment. The equilibrium (of the subgame) of the Vertical Collusion Regime is that agent 1 (good-type, low-cost agent) becomes the winner of the competition, and pays $\Delta_2 > 0$ to the supervisor, and the supervisor makes the truth telling report $\hat{\theta} = \theta$. We can consider several games which will result in this outcome.⁹

Now let us formulate the problem of the principal in this regime. In equilibrium, the supervisor gets $\lambda\Delta_2$. Since the principal takes into account the deadweight cost accompanying the side trade, her problem takes the form of the maximization of the (gross) total surplus minus $(1 - \lambda)\Delta_2$.

Let (a^{VC}, e^{VC}) maximizes the net total surplus

$$TS^{VC} \equiv TS - (1 - \lambda)\Delta_2 = \{f(a) - (\beta - e - k)a - \psi(e)\} - (1 - \lambda)\Delta_2$$

subject to $\Delta_2 > 0$ for parameters k, λ, μ where $\Delta_2 = V_2 - (1/\lambda)(\psi(e + \Delta e) - \psi(e))$.

Then, (a^{VC}, e^{VC}) provides the solution to the first order conditions

$$\left. \begin{aligned} f'(a) - (\beta - e - k) &= (1 - \lambda) \frac{\partial \Delta_2}{\partial a} \\ a - \psi'(e) &= (1 - \lambda) \frac{\partial \Delta_2}{\partial e} \end{aligned} \right\} \Leftrightarrow \left(\frac{\partial TS}{\partial a}, \frac{\partial TS}{\partial e} \right) = (1 - \lambda) \cdot \left(\frac{\partial \Delta_2}{\partial a}, \frac{\partial \Delta_2}{\partial e} \right)$$

$$\text{where } \begin{cases} \frac{\partial \Delta_2}{\partial a} = f'(a) - (\beta - e - k) + 2\mu \\ \frac{\partial \Delta_2}{\partial e} = a - \psi'(e) - \frac{1}{\lambda}(\psi'(e + 2k) - \psi'(e)) \end{cases}$$

Proposition 2: The optimal equilibrium vertical collusion regime has the property that the principal designs the low-powered job $0 < a^{VC} \leq a^F$ for the agent and the high-powered job $e^{VC} \geq e^F$ for the supervisor.

⁸ The bad (high-cost) agent is willing to pay all of Δ_2 to the supervisor in order to obtain the good task, because he anticipates that he will not receive a good recommendation unless he does so. This makes the value of the supervisor's outside option when bargaining with the good agent equal to the value of $\Delta_2' = \lambda\Delta_2$ under vertical collusion with the bad agent. This coincides in essence with the analysis of transaction cost models of vertical integration in multilateral settings by Bolton and Whinston (1993). Suzuki (2005) extends it in a fully non-cooperative set-up.

⁹ For more details, see Suzuki (1999). A simple example is the second-price auction between the two agents.

Proof See Appendix3

Rationale

If a is perturbed around 0, the marginal surplus is induced by $f'(0) - (\beta - e - k)$. However, a deadweight cost is also brought about by $(1 - \lambda)[f'(0) - (\beta - e - k) + 2\mu]$.

Hence, a positive activity is induced, if and only if the following condition holds

$$f'(0) - (\beta - e - k) > (1 - \lambda)[f'(0) - (\beta - e - k) + 2\mu]$$

$$\Leftrightarrow \lambda[f'(0) - (\beta - e - k)] > (1 - \lambda)2\mu.$$

This condition is rewritten as $f'(0) - (\beta - e - k) > (1/\lambda - 1)2\mu$, whose sufficient condition is $f'(0) > (\beta - k) + (1/\lambda - 1)2\mu$. Since the condition which assures $a^F(k) > 0$ for any $k > 0$ is $f'(0) > \beta - k$, the above condition is stronger than the first best case in that $f'(0)$ is required to be greater by $(1/\lambda - 1)2\mu$. This implies that since the positive activity $a > 0$ brings about the deadweight cost $\Delta_2 > 0$, the more positive surplus $f'(0) - (\beta - k)$ is required at $a = 0$ to induce $a > 0$. We easily see that as $f'(0)$ is greater, as λ is greater, and as μ is smaller, the above sufficient condition tends to hold.

In other words, activity a imposes a first-order-loss $(1 - \lambda)2\mu$ due to an inefficient side transfer, namely, rent-seeking for promotion. This term has a discrete negative effect on the overall efficiency, which may dominate the marginal increase of the surplus. For example, when λ is small, the deadweight cost accompanying the side trade is *discrete*, and a discrete jump with a magnitude of $(1 - \lambda)2\mu$ occurs at $a = 0$. Since the total surplus function is *concave* in a , a marginal increase in a around zero generates a positive marginal effect $\lambda[f'(0) - (\beta - e - k)]$ when λ is small. Hence, if the positive effect is dominated by the negative effect evaluated at $a = 0$, no positive activity is induced, and this is not an outcome of equilibrium collusion regime.

Next, we see that sticking to the first best solution (a^F, e^F) will result in the third best outcome under the equilibrium collusion. With a similar logic to the standard agency theory, the principal marginally changes the incentive schemes, and tries to increase the total surplus. As a result, the first best solution (a^F, e^F) cannot be attained, and the principal designs the low-powered job $0 < a^{VC} \leq a^F$ for the agent and the high-powered job $e^{VC} \geq e^F$ for the supervisor, which generate the second best efficiency. The logic in this part is the trade-off between the marginal decrease in efficiency by the change of the optimal activities and the direct gain by a discrete reduction in deadweight cost $-(1 - \lambda)\Delta_2$.

3.3 Equilibrium Lateral Collusion

In this section, we investigate the possibility that two agents bargain bilaterally (laterally) so that the competition (conflict) for vertical collusion only defines *the status quo of the bargaining game* among them.

Two-Stage Problem

1. Given the main contract (a, e) , the two agents bargain over the lateral collusive agreement, under the threat of vertical collusion.

First, the equilibrium payoff vector as the threat point is

$$(\bar{U}_1, \bar{U}_2) = (U(a, e; k) - \Delta_2, U(0; -k))$$

This is equivalent to the equilibrium payoff allocation, which two agents can get in the vertical collusion as the threat point of the lateral collusion regime. Next, since the sum of the payoffs of the two agents in the success of lateral collusion is $U(a, e; k) + U(0; -k)$, the collusive surplus is Δ_2 . This is due to the fact that two agents can increase the sum of their payoffs *by stopping (the rent seeking) competition*. Then, the problem facing a group of collusive agents is to decide the selection of a sole bidder and the appropriate side payment among them. Needless to say, the sole bidder is the good agent, because it maximizes the collusive surplus.

We adopt the Nash bargaining problem as a renegotiation form among the agents. Then, it is formulated as the following maximization of the Nash Product on the bargaining frontier.

$$\begin{aligned} (U_1^*, U_2^*) = \arg \max_{U_1 \geq \bar{U}_1, U_2 \geq \bar{U}_2} & (U_1 - (U(a, e; k) - \Delta_2))(U_2 - U(0; -k)) \\ \text{s.t.} \quad & U_2 = -\lambda(U_1 - U(a, e; k)) + U(0; -k) \end{aligned}$$

where $\Delta_2 = V_2 - (1/\lambda)(\psi(e + 2k) - \psi(e))$ and Δ_2 is the equilibrium side transfer (from agent 1 to the supervisor) *when the vertical collusion occurred*, given the main contract (a, e) .

The Nash Solution is, from the simple computation,

$$(U_1^*, U_2^*) = (U(a, e; k) - \Delta_2/2, U(0; -k) + \lambda\Delta_2/2)$$

Hence, the sum of the two agents' payoffs according to the Nash Solution is

$$U_1^* + U_2^* = U(a, e; k) + U(0; -k) - (1 - \lambda)\Delta_2/2$$

In other words, the two agents first attain an efficient point by stopping competition, then the good (low-cost) agent 1 transfers a side payment $\Delta_2/2$ to the bad (high-cost) agent 2, who actually receives $\lambda\Delta_2/2$, due to the transaction cost. Thus, the total surplus is reduced by $(1 - \lambda)\Delta_2/2$, as the above formula shows.

2. Now let us consider the optimal design of the initial (main) contract. The principal, rationally expecting the above renegotiation among agents, designs the initial (main) contract so as to maximize the total surplus for the two agents, which in this model corresponds to the efficiency.¹⁰

Suppose that (a^{LC}, e^{LC}) maximizes the total surplus

$$TS^{LC} \equiv TS - (1 - \lambda)\Delta_2/2 = \{f(a) - (\beta - e - k)a - \psi(e)\} - (1 - \lambda)\Delta_2/2$$

for parameters k, λ, μ . Then (a^{LC}, e^{LC}) provides the solution to a series of equations

¹⁰ This is not the case, if we interpret $1 - \lambda$ as a (unit) cost for employing an enforcer of side contract. See the footnote 6.

$$f'(a) - (\beta - e - k) = \frac{1}{2}(1 - \lambda) \frac{\partial \Delta_2}{\partial a}$$

$$a - \psi'(e) = \frac{1}{2}(1 - \lambda) \frac{\partial \Delta_2}{\partial e}$$

Proposition 3:

(a) Lateral Collusion improves the overall efficiency, relative to Vertical Collusion, due to (1) the effect brought about by the change in the total payoffs through the change in marginal

incentives (a^{LC}, e^{LC}) , and (2) the deadweight cost being halved $\frac{1}{2} \times (1 - \lambda) \Delta_2^{LC}$.

(b) The difference in deadweight cost between these two regimes is ¹¹

$$(1 - \lambda) \left[\Delta_2^{VC}(a^{VC}, e^{VC}) - \frac{1}{2} \Delta_2^{LC}(a^{LC}, e^{LC}) \right]$$

Renegotiation among the agents modeled as a take-it-or-leave-it offer (TIOLI offer)

If the renegotiation among the agent is modeled as a take-it-or-leave-it offer (TIOLI offer) of a side payment from the good agent 1 to the bad agent 2, it will be zero or a sufficiently small, positive amount $\varepsilon > 0$. If agent 2 rejects the offer $\varepsilon > 0$, he can obtain *nothing* above $U(0; -k)$.

Hence, he will always accept the offer $\varepsilon > 0$. Thus, we can reach the following corollary.

Corollary 3:

(a) If the renegotiation among the agents is modeled as a take-it-or-leave-it offer (TIOLI offer) of a side payment from the good agent 1 to the bad agent 2, the principal can attain the first best efficiency approximately.

(b) The difference in deadweight cost between the Lateral Collusion with this TIOLI offer renegotiation and the Vertical Collusion regime is

$$(1 - \lambda) \left[\Delta_2^{VC}(a^{VC}, e^{VC}) - \varepsilon \right] \rightarrow (1 - \lambda) \Delta_2^{VC}(a^{VC}, e^{VC}) \text{ as } \varepsilon \rightarrow 0$$

Remark

What happens if the renegotiation among agents is modeled as a take-it-or-leave-it offer (TIOLI offer) from the bad agent 2 to the good agent 1? In a vertical collusion regime as a “threat point” of the lateral collusion regime, the good agent 1 obtains $\Delta_1^{LC} - \Delta_2^{LC}$, and the bad agent 2 obtains 0, when $\Delta_2^{LC} \geq \Delta_1^{LC} / 2 \Leftrightarrow \Delta_2^{LC} / \Delta_1^{LC} \geq 1/2$, that is, the supervisor’s outside option is binding. Since $\Delta_2^{LC} - (\Delta_1^{LC} - \Delta_2^{LC}) = 2\Delta_2^{LC} - \Delta_1^{LC} \geq 0$, the bad agent 2 offers a side payment of $\Delta_1^{LC} - \Delta_2^{LC}$ to the good agent 1, and can obtain the residual $2\Delta_2^{LC} - \Delta_1^{LC}$.¹² In this case, the bad agent will obtain the good task, and inefficient task assignment will occur in equilibrium.

3.4 Implications of the equilibrium collusion regimes

Here, let us consider the implication of lateral collusion among the agents. In the vertical collusion, the two agents compete for a favorable supervisory report and the winner of the

¹¹ The gross total surplus $f(a) - (\beta - e - k)a - \psi(e)$ would be greater in the lateral collusion regime.

¹² Maskin (2003) presents a numerical example with the same essence.

competition gets the good task. In the lateral equilibrium collusion, however, the two agents cooperate regarding the problem of task assignment. Though *the principal loses control* over the agent's task allocation in both regimes, in lateral collusion the principal delegates the authority of task assignment to the agents. And, the deadweight cost of a side trade or the cost of a control loss is reduced by one-half compared to the vertical collusion case. Therefore, when $\lambda \in (0,1)$, the second best mechanism is either the collusion-proof mechanism or lateral equilibrium collusion, but **not** vertical collusion.

We interpret these two equilibrium collusion regimes as follows. In the vertical collusion regime, two divisions compete for the good task, because it brings about an additional reward to the party who is assigned the good task. This leads to inefficient *rent-seeking* and associated *bargaining costs*. For example, imagine that divisional managers tempt the supervisor (product manager) by playing golf, which *wastes* time and money. This means the conflict between two divisions carries an associated cost. On the other hand, lateral collusion can be understood as cooperation between two divisions. In this case, side payments may be interpreted, for example, as an invitation of the rival manager to wining and dining, an option which is *less expensive* than the vertical collusion case. In good performance firms, divisions intercommunicate and coordinate their actions based on shared information. This would result in a lower cost (inefficiency) to firms, giving them a competitive advantage.¹³

4. Comparison of Organizational Forms: The solution to the overall problem

Since competition between the agents (units, plants, and divisions) for collusion with the supervisor is a *source of inefficiency*, mitigating it is key in both regimes (Collusion-proof and Equilibrium collusion regimes), an idea that is in line with the value maximization principle.

We want to identify the situations under which one incentive scheme is more viable than the other. However, it is difficult to say anything definitive about which regime is optimal, because this involves making a *global comparison* among the second best outcomes of the various structures. We can, however, reach a definitive conclusion in certain extreme cases.

Proposition 4: Characterization of the Optimal Solution

- (a) For $k > 0$ and $\mu > 0$ fixed, the overall equilibrium induces the optimal equilibrium collusion regime (a^{EC}, e^{EC}) for any λ sufficiently close to 1, where $EC \in \{VC, LC\}$. The incentive schemes are approximately the first best ones. Especially, when $\lambda = 1$, EC attains the exact first best efficiency in both the VC and LC regimes.
- (b) For a $\lambda \in (0,1)$ fixed, the overall equilibrium induces the optimal collusion-proof regime (a^{CP}, e^{CP}) for any $k > 0$ small enough. In this case, the absence of side transfers is optimal, and thus the principal prevents the two agents from competing for the good task.

Economic Rationale on the Results

- (a) Note that $1 - \lambda$ means the marginal deadweight cost resulting from the side payment. Thus, when λ is close to 1, the principal can attain collusion-free first best efficiency approximately,

¹³ Milgrom (1988) and Milgrom-Roberts (1992) studied ‘influence processes’ in firms and how they are optimally managed, i.e. the efficient organization design problem. Their approach is very close to this paper, but they do not deal with the ‘collusion’ problem, at least with regard to “hidden-characteristics” situations.

because the deadweight cost $(1-\lambda)\Delta_2$ tends to zero, while he cannot do so in the collusion-proof scenario, due to the cost of the incentive constraint. Hence, the equilibrium collusion regime is optimal. Especially, *VC* corresponds to the situation where *relevant information is induced through competition between the agents*. *LC* also attains the approximate first best efficiency, and almost only affects the distribution of surplus among parties.

(b) By fixing $\lambda \in (0,1)$, when k tends to zero, the collusion proof (no side-transfer) regime is optimal. In this case ($k \rightarrow 0$), we know that the low-powered incentive scheme for the agent is adopted as the solution. To show the theoretical intuition behind this result, it suffices to take the derivative of the constrained optimum problem with respect to a . If the principal adopts the first best job design, she will obtain a *second-order gain* through changing the optimal behavior. However, the cost through collusion incentive constraints (CIC) is *discretely large*: in other words, the CIC may be violated. Thus, the low-powered and high-powered job designs $a^{CP} \leq a^F$ and $e^{CP} \geq e^F$ are better than the first best job levels a^F and e^F , because the second order distortions due to the marginal changes in incentives are dominated by the first-order gain through relaxing the collusion incentive constraints (CIC). Especially, when k tends to zero, the former second order distortion will be small enough, and so the first best efficiency will be approximately attained.

5. Interpretations of the Model and the Results: Paper's Contribution

5.1 Implications of Two Regimes: Collusion-Proof vs. Equilibrium Collusion

If the principal selects the collusion-proof regime, he does not effectively use the supervisory 'report' or 'message' ($\hat{\theta}$), but allows the two agents to *directly reveal his type subject to the collusion-proof constraint*. In other words, the principal substantially allows the two agents to reveal their type information *without supervisory discretion*, even with the cost of incentive compatibility.

If the principal selects the collusion-allowing regime as the vertical collusion, the initial contracts will be (secretly) renegotiated by the supervisor and the two agents. In this regime, the principal appoints the supervisor to be the referee of the competition between the two agents, thereby delegating the real authority¹⁴ for selection of two agents to the supervisor. Since this is publicly known by both agents, a kind of 'yardstick competition' or 'tournament' is generated between the two agents.¹⁵ However, such delegated decision-making generates a cost of $(1-\lambda)\Delta_2$ in equilibrium, which implies in its essence *the cost of control loss*.

As a theoretical point, as the above results suggest, *the equivalence principle or the collusion-proofness principle proposed by Tirole (1986, 1992) that tells us that we can just focus on collusion proof contracts, does not hold*, over a broad range of parameters,¹⁶ under some

¹⁴ See Aghion-Tirole (1997). In our model, the formal authority allocation structure for the determination of the task assignment is given (exogenous), but the allocation of the real authority is optimally (endogenously) determined.

¹⁵ We could say that our paper results from mixing the yardstick competition (tournament) model ala Lazear-Rosen (1981) and Nalebuff-Stiglitz (1983) with the collusion model ala Tirole (1986, 1992) and Laffont-Tirole (1991).

¹⁶ Suzuki (1999) provides a full characterization of the sets of parameters under which these contractual regimes (Collusion-proof vs. Equilibrium Collusion) emerge in equilibrium.

limitation of the set of the contractual arrangements and a one-principal, one-supervisor, two-agent scenario.

If we allow lateral collusion (side contracting among agents), then the second best efficiency can be increased discretely, *not marginally*: in other words, to a greater extent than the vertical collusion. This result is analytically clear, but there is a large difference in *the amount of shared information* between the two equilibrium collusion regimes. In the lateral collusion, *the detailed observability about each other's actions (cheating behavior from the lateral collusive agreement) plays a crucial role* in its enforcement. Though this is equivalent to adding a different source to the vertical collusion regime, in the sense that not only the incentive scheme but also the information structure induces the higher efficiency (surplus) discretely, this represents a new theoretical consideration in these collusion models with incomplete contracting.

In both the collusion-proof and equilibrium collusion regimes, the low-powered incentive scheme for the agent and the high-powered incentive scheme for the supervisor could be derived as an optimal solution for each of the two regimes. This is consistent with the empirical finding that some well-performing firms avoid the conflict for the good task by adopting “egalitarianism (an extreme kind of low-powered scheme)”, while the firms that fail to do so often suffer from inefficient competition for the good task¹⁷.

5.2 Interpretations: Organization Structure and Authority Delegation

Next, we provide some interpretations of the results in terms of the organization structure and authority delegation. They can, as a whole, be understood as examples of *institutional devices* that reduce the source of inefficiency (waste), in line with the value maximization principle.

As for the organization structure, we can interpret the collusion-proof regime as *the centralized firm*, where the principal effectively allocates the tasks to the divisions. Similarly, we can interpret the lateral equilibrium collusion as *the decentralized firm*, where two divisions negotiate the task assignment and the side payment efficiently among themselves. A side payment is made to the high-cost (inefficient) agent from the low-cost (efficient) agent, and is interpreted such that the low-cost and high-cost divisional managers have a meeting, with *less expensive* drinks and food, and reach an agreement such that the high-cost agent gives up the good task, and in return is rewarded in the form of a payment equivalent to the meeting price (by the low-cost division) and/or some contribution to the good task through a personal dispatch.

This classification corresponds in essence to the terminology provided by Hart and Holmstrom (2002), *Centralized Firm (CF)* and *Decentralized Firm (DF)*, since in both regimes the principal holds the formal and real decision right on the design of an initial contract, while in the CF regime she *actually* makes the task assignment decision through the collusion-proof constraint, that is, makes both decisions on *contract design* and (*real*) *task assignment*, and in DF regime(s), she delegates the *actual* task assignment decision, which is a *non-contractible* action, to the supervisor (in the vertical collusion regime) or to the two agents (in the lateral collusion regime).

Following the terminology of Aghion and Tirole (1997), the ex ante contract in each regime (collusion-proof solution, equilibrium with vertical collusion and equilibrium with lateral collusion) has economic implications regarding *real* authority delegation, as well as the *formal* allocation of authority. As Aghion and Tirole (1997) stress, *formal* authority means who has the *right* to make the decision on the task assignment¹⁸, which in our model is held by the principal in

¹⁷See, Takamiya (1980).

¹⁸The task assignment decision itself can be interpreted as a *non-contractible action* and determined as an equilibrium outcome of a hidden-gaming/bargaining game between the supervisor and the two agents. This is a setting similar to that presented by Aghion -Tirole (1997), where the principal and the agent exert themselves in an effort to acquire

the three regimes, and importantly this differs from *real authority*, that is, who *actually* makes the decision on the task assignment. In the collusion-proof regime, the real authority over the task assignment problem is held by the principal. She actually makes decisions about the human resource allocation problem, *through imposing collusion-proof constraints*. In the equilibrium vertical collusion regime, the real authority is held by the supervisor (intermediary). Hence, in such cases, the incentive to collude with the supervisor and manipulate the information (or try to deter it) would be very strong. Finally, in the lateral collusion regime, the two agents *jointly* have the real authority over the task assignment problem. They communicate, come to a common understanding, and decide on a solution to the task assignment problem through mutual consent. In this case, the *ex ante* competing behavior (conflict) for the good task would be largely mitigated, leading to the good performance (competitiveness) of the firm. In summary, we can say that the principal has the formal authority --- the *right* to decide on the task assignment --- but she is uninformed about which agent is better and so will ask the other parties for a (substantial) recommendation and follow it if the cost of delegation is relatively low. In that case, *real* authority is delegated to parties other than the principal, and then the principal commits herself to *not* deciding the task assignment on her own.

6. Applications

Here we consider some directions for the application or extension of our theoretical model.

1. A study of “the dark side of internal capital markets” by Scharfstein and Stein (2000). They consider a three-tier agency model, consisting of outside investors, a CEO and two divisional managers, and show how rent-seeking behavior on the part of divisional managers can undermine the workings of an internal capital market. By rent-seeking, divisional managers can raise their bargaining power and extract greater compensation from the CEO. And because the CEO is herself an agent of outside investors, i.e., a supervisor in our model, such extra compensation may take the form of *preferential* budget allocations. Our model analyzes similar undesirable (inefficient) competition between divisions in a three-tier agency situation with negative externality. The readers could review our model from the viewpoint of ‘winner picking and *preferential* budget allocations among different projects in a firm.

2. Organizational structures of multinational enterprises. As the empirical literature (e.g. Takamiya, 1980; Yasumuro, 2001) describes, there exist conflicts of interests generated by a variety of externalities in multinational enterprises. Takamiya (1980) points out that fierce competition exists for a good task between divisions or subsidiaries in Japanese Europe multinationals, which is very similar to the scenario for our model. Yasumuro (2001) discusses another form of externality between an internal division, international division, and foreign subsidiary, which leads to a hold-up (free rider) problem *in* a firm. Such conflicts of interests (the source of inefficiency) could be solved by adequately choosing the organizational structure (centralization vs. decentralization and authority structure) as well as transfer pricing (“Internalization of externalities”).

3. Political Economy Application. From modeling explicitly the collusion game among the supervisor and two agents, and considering an explicit bargaining process, we can perform a clear-cut cost vs. benefit analysis of political activities, and make clear how the actions of parties lead to the collusion outcome or not through some form of bilateral bargaining. Hamada and Okuno (1992) present a political economy (game theoretic) approach to international trade negotiations. They point out that even though it is more desirable due to a higher degree of internalization of externalities, it is difficult for multi parties to commit to the inclusive negotiation framework (a sort of grand contract), since the two parties have an incentive to

information, and then a decision is made on which project to pursue.

organize bilateral bargaining to Pareto-improve their position, and thereby dilute the framework of multilateral inclusive bargaining. The rule of bargaining is chosen *ex ante* so as to internalize as many externalities as possible, under the possibility of subsequent bilateral bargaining. Our model is close to this idea, and is therefore applicable to political economy contexts.

7. Concluding Remarks

In this paper, we formulated a firm with two divisions consisting of two product development teams with a common supervisor¹⁹ (e.g. product manager) as a four-player game, and modeled a task design/task assignment process. We provided a partial characterization of the equilibrium of this four-player game, and showed which regime improves efficiency between the collusion proof regime and the lateral collusion regime as a second best mechanism, under the assumption that (1) the principal faces an intervention cost when she has less information than the supervisor and the two agents, and (2) the bargaining between the parties is costly. The point is that by coping with the trade-off of the value of information vs. either the cost of the collusion incentive constraint (in the collusion-proof regime) or the rent-seeking (lobbying) cost (in the equilibrium collusion one), the principal may want to let collusion happen in equilibrium, and simultaneously the allocation of (real) authority is endogenously determined.

Two theoretical observations should be made. First, in our model, a side payment is accompanied by the deadweight cost $1 - \lambda$ per unit. This is the cost of bargaining (more generally, the transaction cost in side trades), given *exogenously*. The typical literatures in this field (starting from J.Tirole, 1986 and 1992) have assumed the *exogeneity* of this transaction cost. However, this will have to be justified. Laffont and Martimort (1997) showed the endogenization of the deadweight cost as generated in the Bayes Nash equilibrium in the collusion game under asymmetric information between the supervisor and the agent. If we consider the cases where only one party among the supervisor and the agents knows the state of nature, e.g., a collusion game with asymmetric information among them, in a perfect Bayesian equilibrium of the game, the bad agent, who has the private information about the situation (that is, the informed player), makes a side contract proposal to the good agent who does not have the information (the uninformed player), concealing his type, then the bad agent will get the good job. In this case, inefficiency would be *endogenously* brought about.²⁰ Also, if we assume that the agents privately know their own types, they negotiate a collusive agreement under asymmetric information. In such cases, it is possible that the wrong vertical collusion might be stochastically formed, and hence the induced efficiency might be lower.

Second, an important theoretical observation should be made about the side contract between the supervisor and the agent. Since this contract is usually non-enforceable, it must be *self-enforcing*. Nonetheless, in this paper, we assume that the parties can communicate with each other and can make a *binding contract* with regard to the outcome of the solution of the task assignment problem. Making collusion endogenous in such multi-layer organizations may be interesting. If their relationship is repeated, their lateral collusion outcome will be sustained in equilibrium by the threat of a vertical collusion outcome, which will also be sustained off the equilibrium path. Instead, if we assume that only the two agents are able to engage in cheap talk without being detected, a cheap talk equilibrium is, by definition, completely self enforcing. It may be found that under certain conditions in relation to the principal's incentive schemes, cheap talk expands the set of equilibrium outcomes, generating *self-enforcing* lateral collusion. Any of several refinement criteria for cheap talk games in the existing literature (e.g. J.Farrell, 1993)

¹⁹ Introducing another supervisor into the model under various information structures and considering multilateral interactions among them (two supervisors and two agents) would be a challenging but interesting extension.

²⁰ Suzuki (1999) derives this outcome as an equilibrium of the modified model.

suggests that only the collusive equilibria survive, supporting the self-enforceability of the lateral collusion. Though we feel it is important to point to the enforcement problem associated with side contracts, our assessment in this paper makes a contribution to the analysis of organizations where the possibility of collusion exists, and the implications from the viewpoint of authority delegation.

APPENDICES

Appendix 1: Proof of Proposition1

Step1 First, we show that the corner solution $a^{CP} = 0$ is not optimal. Suppose that $a^{CP} = 0$. Then, any (\hat{a}, \hat{e}) with $\hat{a} = 0$ always satisfies $\Delta_2 \leq 0$. Then, e^{CP} are the solution to $\max_{\{\hat{e}\}} f(\hat{a}) - (\beta - \hat{e} - k)\hat{a} - \psi(\hat{e})$ s.t. $\hat{a} = 0 \Rightarrow \Delta_2 \leq 0$, and the solution is $\hat{a} = 0, \hat{e} = 0$.

By the way, the optimal solution $\{a^{CP}, e^{CP}\}$ must satisfy the above the Kuhn-Tucker condition. Thus, we check whether a Kuhn-Tucker multiplier ξ exists such that the triple $\{a, e, \xi\}$ satisfies the above FO conditions and the Complementary Slackness Conditions. For $a^{CP} = 0, e^{CP} = 0$, the FO conditions are reduced to

$$\begin{cases} k = \xi(k + 2\mu) \\ 0 = \xi \left(-\frac{1}{\lambda} \psi'(2k) \right) \end{cases}$$

From the first condition, $\xi = \frac{k}{k + 2\mu} > 0$ for $k > 0$ and $\mu > 0$. But then, the second condition does not hold for $\lambda \in [0, 1]$ and $\psi'(2k) > 0$. Therefore we see that a multiplier ξ does not exist such that the triple $\{a^{CP} = 0, e^{CP} = 0, \xi\}$ satisfies the first order conditions. Hence, $\hat{a} = 0, \hat{e} = 0$ is indeed collusion-proof, but not optimal.

Step2 Next, we investigate the inner solution case. From the SOC conditions, we have:

$$\begin{aligned} f''(a) - \xi \frac{\partial^2 \Delta_2}{\partial a^2} &= (1 - \xi) f''(a) < 0 \\ -\psi''(e) - \xi \frac{\partial^2 \Delta_2}{\partial e^2} &= -\psi''(e) - \xi \left[-\psi''(e) - \frac{1}{\lambda} (\psi''(e + 2k) - \psi''(e)) \right] < 0 \end{aligned}$$

Hence, ξ satisfies $0 < \xi < \frac{\psi''(e)}{\psi''(e) + \frac{1}{\lambda} (\psi''(e + 2k) - \psi''(e))} < 1$

Then, evaluating the FO conditions at the first best solution (a^F, e^F) , which satisfies

$$f'(a^F) - (\beta - e^F - k) = 0 \text{ and } a^F - \psi'(e^F) = 0$$

we have $0 - \xi \cdot 2\mu < 0$ and $\xi \left[\frac{1}{\lambda} (\psi'(e^F + 2k) - \psi'(e^F)) \right] > 0$.

Therefore, the principal should design the low-powered job $a^{CP} < a^F$ for the agent and the

high-powered job $e^{CP} > e^F$ for the supervisor.

Q.E.D

Appendix 2: Proof of Corollary1

Proof

Under the Lagrange multiplier ξ when the SOC's hold, a^{CP} and e^{CP} are complements, in that the cross derivative of the Lagrange function with respect to a and e is positive, which can be checked by differentiating each of the FOC's with respect to the other variable. That is, by

differentiating $f'(a) - (\beta - e - k) - \xi \frac{\partial \Delta_2}{\partial a} = 0$ with respect to e , we have $1 - \xi > 0$ ($\xi < 1$

under the SOC's), and similarly by differentiating $a - \psi'(e) - \xi \frac{\partial \Delta_2}{\partial e} = 0$ with respect to a , we

have $1 - \xi > 0$. Then, (a) when μ goes up, $\partial \Delta_2 / \partial a$ goes up, and the optimal a^{CP} goes down. By the complementarity of a^{CP} and e^{CP} , e^{CP} also goes down. Next, (b) when λ goes up, $\partial \Delta_2 / \partial e$ goes up. Thus, the optimal e^{CP} goes down. By the complementarity of a^{CP} and e^{CP} , a^{CP} also goes down.

Q.E.D

Appendix 3: Proof of Proposition2

Step1. We prove that $a^{VC} > 0$ and $e^{VC} > 0$ in the equilibrium vertical collusion regime $\Delta_2 > 0$.

First, when $a = 0$, $\Delta_2 = \{f(0) - (\beta - e - k) \cdot 0 - \psi(e) + 2\mu \cdot 0\} - (1/\lambda)(\psi(e + 2k) - \psi(e))$
 $= -\psi(e) - (1/\lambda)(\psi(e + 2k) - \psi(e)) < 0 \quad \forall e \geq 0$

That is, $\Delta_2 > 0$ does not hold. Hence, we must have $a > 0$ when $\Delta_2 > 0$.

Next, for any $a > 0$, evaluating the first order condition at $e = 0$, we have

$$a - \psi'(0) - (1 - \lambda) \frac{\partial \Delta_2}{\partial e} \Big|_{e=0} = a - 0 - (1 - \lambda) \left(a - 0 - \frac{1}{\lambda} \psi'(2k) \right) = a + \frac{1 - \lambda}{\lambda} \psi'(2k) > 0$$

This means that by marginally increasing e from 0, more efficiency can be induced. Hence, $e^{VC} > 0$ at the optimum.

Step2. Next, we investigate the inner solution. From the SOC conditions, we have:

$$f''(a) - (1 - \lambda) \frac{\partial^2 \Delta_2}{\partial a^2} = \lambda f''(a) < 0$$

$$-\psi''(e) - (1 - \lambda) \frac{\partial^2 \Delta_2}{\partial e^2} = -\psi''(e) - (1 - \lambda) \left[-\psi''(e) - \frac{1}{\lambda} (\psi''(e + 2k) - \psi''(e)) \right] < 0$$

Hence, $\lambda \in [0, 1]$ should satisfy $1 - \frac{\psi''(e)}{\psi''(e) + \frac{1}{\lambda} (\psi''(e + 2k) - \psi''(e))} < \lambda \leq 1$

Then, evaluating the FO conditions at the first best solution (a^F, e^F) , which satisfies

$$f'(a^F) - (\beta - e^F - k) = 0 \text{ and } a^F - \psi'(e^F) = 0$$

we have $0 - (1 - \lambda) \cdot 2\mu \leq 0$ and $(1 - \lambda) \left[\frac{1}{\lambda} (\psi'(e^F + 2k) - \psi'(e^F)) \right] \geq 0$ for the above λ .

Therefore, the principal should design the low-powered job $0 < a^{VC} \leq a^F$ for the agent and the high-powered job $e^{VC} \geq e^F$ for the supervisor. **Q.E.D**

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